LAW OF RANDOM ERRORS.

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LET the unknown error in position of one measured determination of a point in space be δ, then the point sought may be anywhere on the surface of the sphere whose centre is the observed point and radius δ. Therefore, by the definition of probability, the probability that any point on this surface is the one sought is $\frac{1}{4\pi m\delta^2}$, where m is the number of points on a unit of surface.

Since then each random error gives a number of liable positions proportional to its square, therefore the probability that any one of the total number of liable points, resulting from n measured determinations of a point in space, is the one sought, is

$$y = \frac{1}{4\pi m S(\delta^2)},$$

where $S(\delta^2)$ is the sum of the squares of the random errors belonging to the n points given by measurement or observation. Hence of all points resulting from the n unknown errors, that which has the greatest probability is the one which makes $S(\delta^2)$ a maximum, that is, it is at the centre of gravity of the given points, and its probability is

$$p = \frac{1}{4\pi m S(\delta_1{}^2)},$$

where $S(\delta_1{}^2)$ is the sums of the squares of the distances from the given p'ts to their centre of gravity. Hence

$$y = p\frac{S(\delta_1{}^2)}{S(\delta^2)} = \frac{pS(\delta_1{}^2)}{S(\delta_1{}^2) + nx^2} = \frac{p}{1 + \dfrac{nx^2}{S(\delta_1{}^2)}},$$

where x, by problem 200 (ANALYST, Vol. V, page 91), is the distance from any one of the $4\pi m S(\delta^2)$ points to the centre of gravity of the n observed points, and y is the probability, frequency or density of errors at magnitude x. Hence

$$\int_0^x y\,dx = z = p\sqrt{\frac{S(\delta_1{}^2)}{n}}\tan^{-1}\sqrt{\frac{nx^2}{S(\delta_1{}^2)}},$$

where z is the number of errors included by x. Hence

$$n = p\sqrt{\frac{S(\delta_1{}^2)}{n}}\tan^{-1}\sqrt{\frac{nl^2}{S(\delta_1{}^2)}},$$

where l is the magnitude of the error which includes the whole number n.

$$\therefore \frac{z}{n} = \frac{\tan^{-1}\sqrt{\dfrac{nx^2}{S(\delta_1{}^2)}}}{\tan^{-1}\sqrt{\dfrac{nl^2}{S(\delta_1{}^2)}}} = \frac{\tan^{-1}\sqrt{\dfrac{nx^2}{S(\delta_1{}^2)}}}{2\tan^{-1}\frac{1}{2}\sqrt{2}},$$

where $z \div n$ is the probability that an error shall not exceed x. It being proved at page 189, Vol. VII, that the constant

$$\tan\tfrac{1}{2}\tan^{-1}\left(\frac{nl^2}{S(\delta_1{}^2)}\right)^{\frac{1}{2}} = \tfrac{1}{2}\sqrt{2}.$$

Hence

$$x = \sqrt{\frac{S(\delta_1{}^2)}{n}} = \tan\left(\frac{2z}{n}\tan^{-1}\tfrac{1}{2}\sqrt{2}\right).$$

And for $z \div n = \frac{1}{2}$, the probable error is

$$x_1 = \tfrac{1}{2}\sqrt{2}\sqrt{\frac{S(\delta_1{}^2)}{n}}.$$

A line or surface is in its most probable position when the sum of the squares of the normals upon it, from the given positions, is a minimum, which normals are the errors.

In problem 239 (ANALYST, Vol. VI, p. 49), for A, $z \div n = \frac{36}{100}$, and his angular error $x = \tan^{-1}\frac{5}{3600} = \tan^{-1}\frac{1}{720}$. Hence

$$\frac{\text{A's prob. er.}}{\text{B's prob. er.}} = \sqrt{\frac{S(\delta_1{}^2)}{S(\delta_1'^2)}} = \frac{\tan\left(\frac{32}{25}\tan^{-1}\frac{1}{2}\sqrt{2}\right)}{\tan\left(\frac{18}{25}\tan^{-1}\frac{1}{2}\sqrt{2}\right)} \times \frac{\tan^{-1}\frac{1}{720}}{\tan^{-1}\frac{1}{540}} = 1.45.$$

SOLUTION OF A PROBLEM.

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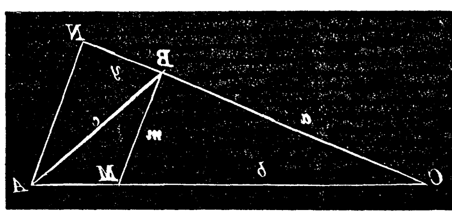
Problem:— In a plane triangle ABC there is given $a+b$, c and m, a perpendicular to BC drawn from B to AC, to solve the triangle.



Solution. From triangle ACN we have

$$b^2 = (a+y)^2 + c^2 - y^2 = a^2 + 2ay + c^2,$$

whence $\qquad 2ay = b^2 - a^2 - c^2 = (a+b)^2 - 2a(a+b) - c^2,$

or $$y = \frac{(a+b)^2 - c^2}{2a} - (a+b) = \frac{k}{a} - n, \qquad (1)$$

where $2k = (a+b)^2 - c^2$ and $n = a+b$.